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Waves in metals at low temperatures

BY P. MISRA*

Department of Physics, Ravenshaw College, Cuttack.

AND

S. K. RAY,

Department of Physics; B. J. B. College, Bhubaneswar

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From the explicit expressions of response function, obtained by Misra (1966), the nature of waves that can propagate inside metals at extremely low temperatures can be analysed in presence of a steady magnetic field. The results so obtained are similar to the experimental results of Bowers *et al* (1961). They reported that at 4°K in presence of a magnetic field of 10,000 gauss waves of frequency 32 cycles/sec. can propagate inside sodium in the direction of the field with a phase velocity of 0.68 cm./sec.

INTRODUCTION

Inside metals, there are positive ions and free electrons. It can be considered that these free electrons are moving on a positive background. Hence these can be treated as electron gas. The density of these electrons is very high i.e., 10^{22} per c.c. Therefore, at low temperature the free electrons inside metals are degenerate in the statistical sense, and the thermal velocity of electrons obey Fermi-Dirac distribution law. Misra (1966) obtained expressions for the response function of an electron gas using Fermi-Dirac statistics. Therefore, his results can be used to study the nature of wave propagation inside metals.

Refractive Index from Response Function :

In the presence of a magnetic field the dielectric function is an unsymmetric tensor and not real even in the absence of absorption. In this case displacement vector is parallel to the electric field only when they are in the direction of the magnetic field, i.e.

$$D_3 = \epsilon_{33} E_3 \qquad \ldots(1)$$

But for other components

$$D_1 \pm iD_2 = (\epsilon_{11} \pm \epsilon_{12})\ (E_1 \pm iE_2) \qquad \ldots(2)$$

From this it is clear that the displacement vector in the XY plane is proportional to the electric field that rotates clockwise or counterclockwise. Therefore we can write from equation (2)

$$D_{lr} = \epsilon_{lr}\ E_{lr} \qquad \ldots(3)$$

*Present Address : Reader in Physics, S. C. S. College, Puri.

Hence ϵ_{lr} can be expressed in terms of the response function as follows :

$$\epsilon_{lr} = 1 - \frac{4\pi i K_{lr}\,(\vec{k},\omega)}{\omega} \qquad \ldots(4)$$

where $K_{lr} = K_{11} \pm iK_{12}$

Using the expressions for response function, obtained by Misra (1966) in equation (4), we get assuming the absorption to be small,

$$\begin{aligned} n^2_{lr} = 1 - \frac{3\omega_0^2 c}{4\omega^2 n^3_{lr} v_0^3} \Bigg[& \frac{2cn_{lr} v_0(\omega \pm \Omega)}{\omega} \\ & + \left\{ n^2_{lr}\, v_0^2 - \frac{c^2(\omega \pm \Omega)^2}{\omega^2} \right\} ln \left[\frac{c(\omega \pm \Omega) + v_0 n_{lr}\, \omega}{c(\omega \pm \Omega) - v_0 n_{lr}\, \omega} \right] \\ & - \frac{\pi^2 \omega_0^2 c}{8m^2\beta^2\omega^2 n_{lr} v_0^4} \Bigg[\frac{4cn_{lr}\omega\,(\omega \pm \Omega)}{c^2(\omega \pm \Omega)^2 - v^2_0 n^2_{lr}\omega^2} \\ & + \frac{1}{v_0} ln \left\{ \frac{c(\omega \pm \Omega) + v_0 n_{lr}\, \omega}{c(\omega \pm \Omega) \quad v_0 n_{lr}\, \omega} \right\} \Bigg] \end{aligned} \qquad \ldots(5)$$

where we have taken $k = \frac{n_{lr}\,\omega}{c}$

Propagation of electromagnetic waves in a metal can be analysed with the aid of equation (5). But this expression is very lengthy and complicated. However, for the two following special cases, the wave propagation can easily be analysed,

Case I $\qquad \dfrac{v_0 \omega n_{lr}}{c(\omega \pm \Omega)} < 1,$

Case II $\qquad \dfrac{c\,(\omega \pm \Omega)}{v_0 \omega n_{lr}} < 1.$

The first case is satisfied for $n_{lr} < 1$ and $\omega > \Omega$. Under this approximation we can expand

$$ln \left| \frac{1 + \dfrac{v_0 n_{lr}\omega}{c(\omega \pm \Omega)}}{1 - \dfrac{v_0 n_{lr}\omega}{c(\omega \pm \Omega)}} \right| \quad \text{and} \quad \left[1 - \frac{v_0^2 n_{lr}^2 \omega^2}{c^2(\omega \pm \Omega)^2} \right]^{-1}$$

in a power series of $\dfrac{v_0 n_{lr}\omega}{c(\omega \pm \Omega)}$ and neglect terms much less than one.

After simple calculation we get

$$n_{t\,r}^{2} = \frac{1 - \dfrac{\omega_0^{\,2}}{\omega(\omega \pm \Omega)}\left[1 + \dfrac{3\pi^2}{4m^2\beta^2 v_0^{\,4}}\right]}{1 + \dfrac{\omega_0^{\,2} v_0^{\,2}\omega}{5c^2(\omega \pm \Omega)^3}\left[1 + \dfrac{35\pi^2}{12m^2\beta^2 v_0^{\,4}}\right]} \qquad \ldots(6)$$

The second case is satisfied for $\omega \sim \Omega$ as well as for ω, for which $n_{t\,r}^{\,2} \gg 1$. Under this approximation we get,

$$n_{t\,r}^{\,2} = 1 - \frac{3\omega_0^{\,2}c^2(\omega \pm \Omega)}{\omega^3 v_0^{\,2} n_{t\,r}^{\,2}}\left[1 - \frac{\pi^2}{12m^2\beta^2 v_0^{\,4}}\right] \qquad \ldots(7)$$

Discussion of the Results

From equation (6) it is clear that for $\omega_0^{\,2} > \dfrac{1 + \dfrac{3\pi^2}{4m^2\beta^2 v_0^{\,4}}}{\omega(\omega \pm \Omega)}$ refractive index is imaginary. So the wave cannot propagate through the medium. For $\omega > \omega_0$ equation (6) becomes

$$n_{t\,r}^{\,2} = 1 - \frac{\omega_0^{\,2}}{\omega(\omega \pm \Omega)}\left[1 + \frac{3\pi^2}{4m^2\beta^2 v_0^{\,4}}\right]$$

Further if we neglect the effect of temperature the above equation reduces to AH equation, when the wave is propagating in the direction of the magnetic field.

Equation (7) shows that $n_{t\,r}$ is always imaginary for the ordinary wave. Hence the ordinary wave cannot propagate through the medium for $n_{t\,r} > 1$, whereas, the extra-ordinary wave can propagate through the medium for values of ω less than Ω. To get a clear picture of electromagnetic waves that can propagate through sodium at 4°K, the calculated values of $n_{t\,r}^{\,2}$ for different values of ω are given in the table. In the calculation, the magnitude of the steady magnetic field is taken to be 10^4 gauss and density of free electrons to be 2.6×10^{22}.

Table

ω	n^2_{tr}	ω	n^2 imaginary
10	1.3×10^{42}	1.8×10^{11}	
60	8.3×10^{20}	9.12×10^{15}	0
192	1.14×10^{20}	1×10^{16}	0.27
10^6	1.3×10^{16}	1.5×10^{16}	0.63
10^7	13×10^{12}	2×10^{16}	0.8
10^9	6×10^9	3×10^{16}	0.91
1.5×10^{11}	8×10^6	4×10^{16}	0.95
1.75×10^{11}	4×10^5	1×10^{17}	0.99
Ω	1		

From the table it is clear that in case of sodium, electromagnetic waves, for which, $9.12 \times 10^{15} > \omega > 1.76 \times 10^{11}$, can not propagate through. But it is interesting to note that even waves of frequency of the order 10 cycles/sec. can penetrate through sodium in the direction of the magnetic field. The phase velocity of the wave is of the order of 1 cm. Similar qualitative experimental results were obtained by Bowers *et al* (1961). From their results it comes out that electromagnetic waves of 32 cycles per second can penetrate through sodium in the direction of the magnetic field of magnitude 10^4 gauss at 4°K. And the refractive index is found to be 1.5×10^9.

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